

Question 6

Let $f(x) = |x - 2| + |x + 1|$.

- (a) Express $f(x)$ as a piecewise-defined function.
- (b) Sketch the graph of $f(x)$.
- (c) Determine all values of x where $f(x)$ is not differentiable. Justify your answer.

Question 6 - Solution

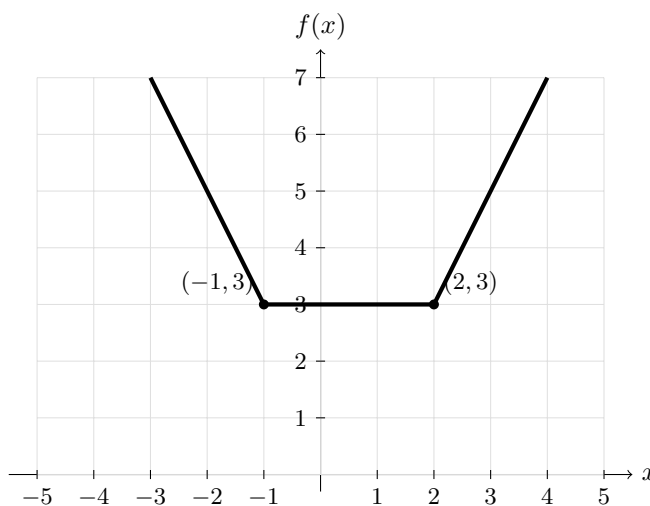
(a) The absolute values change sign at $x = -1$ and $x = 2$.

$$f(x) = \begin{cases} -(x-2) - (x+1), & x < -1, \\ -(x-2) + (x+1), & -1 \leq x < 2, \\ (x-2) + (x+1), & x \geq 2. \end{cases}$$

After simplifying,

$$f(x) = \begin{cases} -2x + 1, & x < -1, \\ 3, & -1 \leq x < 2, \\ 2x - 1, & x \geq 2. \end{cases}$$

(b) The graph decreases until $x = -1$, stays constant from $x = -1$ to $x = 2$, and then increases.



(c) The only possible non-differentiable points are where the absolute value expressions change sign:

$$x = -1 \quad \text{and} \quad x = 2.$$

From the piecewise function, the slopes are

$$\begin{cases} -2, & x < -1, \\ 0, & -1 < x < 2, \\ 2, & x > 2. \end{cases}$$

At $x = -1$, the slope changes from -2 to 0 , so f is not differentiable there.

At $x = 2$, the slope changes from 0 to 2 , so f is not differentiable there.

Therefore,

$$\boxed{x = -1, 2}$$