

Question 2

A piecewise function is defined as follows:

$$f(x) = \begin{cases} x^2 - 4x + 3 & \text{if } x < 2 \\ \sqrt{x+2} & \text{if } x \geq 2 \end{cases}$$

- (a) Determine whether $f(x)$ is continuous at $x = 2$.
- (b) Determine whether $f(x)$ is differentiable at $x = 2$.

Question 2 - Solution

Part (a): Continuity at $x = 2$ We must check:

- $\lim_{x \rightarrow 2^-} f(x)$
- $\lim_{x \rightarrow 2^+} f(x)$
- $f(2)$

For $x < 2$, use $f(x) = x^2 - 4x + 3$:

$$\lim_{x \rightarrow 2^-} f(x) = 2^2 - 4(2) + 3 = 4 - 8 + 3 = -1$$

For $x \geq 2$, use $f(x) = \sqrt{x+2}$:

$$\lim_{x \rightarrow 2^+} f(x) = \sqrt{2+2} = \sqrt{4} = 2$$

Since:

$$\lim_{x \rightarrow 2^-} f(x) = -1, \quad \lim_{x \rightarrow 2^+} f(x) = 2$$

the limit from left and right are not equal.

Conclusion: $f(x)$ is **not continuous** at $x = 2$

Part (b): Differentiability at $x = 2$

A function must be continuous at a point to be differentiable there. Since $f(x)$ is not continuous at $x = 2$, it cannot be differentiable there.

Conclusion: $f(x)$ is **not differentiable** at $x = 2$