

Question 1

Let $f(x) = \sqrt{x+4}$ and $g(x) = \frac{1}{x-2}$. Define the composition $h(x) = f(g(x))$.

Find the domain of $h(x)$, and express your answer in interval notation.

Question 1 - Solution

We are given:

$$f(x) = \sqrt{x+4}, \quad g(x) = \frac{1}{x-2}, \quad h(x) = f(g(x)) = \sqrt{\frac{1}{x-2} + 4}$$

To determine the domain of $h(x)$, we consider the restrictions from both the inner and outer functions.

Step 1: Domain restriction from $g(x)$

The function $g(x) = \frac{1}{x-2}$ is undefined when the denominator is zero. This occurs at $x = 2$, so:

$$x \neq 2$$

Step 2: Domain restriction from the square root

We need the expression inside the square root to be greater than or equal to zero:

$$\frac{1}{x-2} + 4 \geq 0 \Rightarrow \frac{4x-7}{x-2} \geq 0$$

Step 3: Critical points and sign analysis

$$4x - 7 = 0 \Rightarrow x = \frac{7}{4}, \quad x - 2 = 0 \Rightarrow x = 2$$

From the sign chart:

Positive on $(-\infty, \frac{7}{4}]$, Negative on $(\frac{7}{4}, 2)$, Positive again on $(2, \infty)$.

Final Domain:

$$\boxed{(-\infty, \frac{7}{4}] \cup (2, \infty)}$$